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Structural change in tail behaviour and the recent financial crises

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Abstract: This paper is a contribution in exploring the empirical evidence of the instability in the tail behaviour returns of stock market indices, based on some developments in the analysis of structural change models. The proposed approach can jointly determine the number of structural breaks in a series of tail-indexes and estimate the mean tail-index levels in different regimes. Here we advocate a modified Hill estimator for the tail index. We provide simulations that indicate good finite sample properties of our procedure. The proposed method is then applied to the tail behaviour returns of two international stock market indices, SP500 (USA) and CAC40 (France). The results indicate that procedures perform reasonably well and lead to an appropriate number of breaks with locations coinciding with major financial crisis and events such as the LTCM crisis, September 11, 2001 terrorist attack, sub-prime crisis in 2008 and European Union (EU) debt crisis triggered on 2010.

Keywords: multiple structural change; extreme value analysis; tail-index; modified-hill estimator, break dates.

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1 Introduction

Recent financial disasters and crises have strengthened academic attention to the extremal behaviour of financial market returns in turbulent periods, where monitoring financial risk has become a paramount importance for financial institutions as well as capital market regulators. Firms perform risk management to guard against the risk of loss due to the fall in prices of financial assets held or issued by the company. What are of importance here are the magnitudes of the changes in prices, rather than the average variations. Financial risk management typically deals with such low probability events in the tails of asset return distributions. The further out in the tails we go, the smaller the probability of an event becomes at the same time as its consequences will be larger. In order to model such events Extreme Value Theory (EVT) and especially heavy tailed distributions play a crucial role. By heavy tailed distributions, we mean the distribution that have a higher density than that what is predicted under the assumption of normality. For example, a distribution that has an exponential decay (as in the normal) or a finite endpoint is considered thin tailed, while a power decay of the density function in the tails is considered a fat tailed distribution. Examples of such distribution are frequently encountered in many fields, especially in finance and economics (Mandelbrot, 1963; Fama, 1965; Jansen and de Vries, 1991; McNeil, 1998; Embrechts et al., 1999; Gençay and Selçuk, 2004; Huang and Lin, 2004; So and Yu, 2006; Ané, 2006; Cheong, 2008; Marimoutou et al., 2009).

Recent developments in Extreme Value Theory (EVT) enable such fat-tailed data to be analyzed without explicit assumptions having to be made about the distribution of returns. A tail index can be used to measure the fatness of the tails. This way of treatment is originated from the assumption that the tails follow power-like distribution or regularly varying distribution (Embrechts et al., 1997). The rate of decay is thus represented by tail index, which is the inverse of shape parameter. That is, the tail behaviour is governed by tail index. A variety of procedures to estimate are now available in the literature (see Hill, 1975; Pickands, 1975; de Haan and Resnick, 1980; Hall, 1982; Mason, 1982; Davis and Resnick, 1984; Csoérgo, et al., 1985, Hall and Welsh, 1985), although there are still open problems. Quite often the accuracy of these estimators relies heavily on the choice of some threshold, but it is not our aim here to address this type of optimality questions.

The tail index is a necessary ingredient and once the tail index is known it can be used by risk managers or financial regulators to calculate extreme quantiles (like Value at Risk) for very low corresponding significance levels. In this paper, we concentrate on the best known estimator, Hill's estimator, which is commonly accepted and practiced as a successful tool. Numerous empirical studies subsequently focused on identifying the degree of probability mass in the tail by estimating the so called tail index. However, much less attention has been paid to the possibility and consequences of a non-constant tail index. What is important in analyse of tail behaviour, especially interesting during periods of market turbulence, is whether the tail shape of the distribution itself changes, thereby increasing (or decreasing) the probability of outliers. From a risk management point of view, it is crucial for risk manager to know whether the tail shape of financial returns distribution exhibits multiple structural changes, namely, whether the probability of extreme events varies among different regimes. In fact, if the amount of probability mass in the tails of the unconditional distribution is shifting through time, the full sample estimates of tail index and corresponding quantiles might wrongly estimated.

With reference to the existing literatures, only a few empirical studies that have paid attention to the issue of whether the tail index is constant over time. Among the few pioneers studies, are the works of Phillips and Loretan (1990), Koedijk et al. (1990), Jansen and de Vries (1991) or Pagan and Schwert (1990). These empirical literatures on the constancy issue mainly focus on testing for a single known (and thus exogenously selected) breakpoint in α and provide only weak evidence for structural change in tail behaviour.

Recently, some works that focus on the subject have been emerged. Examples include Quintos et al. (2001), Werner and Upper (2004) and Candelon and Straetmans (2006). Quintos et al. (2001) proposed tests statistics that allow for the identification of single but unknown (endogenously determined) breakpoints in α. They applied a recursive, rolling and sequential test statistic to the tails of three emerging stock markets and were able to detect structural breaks indeed. Moreover, the detected breakpoints were found to be 'meaningful' in the sense that they coincided with periods of financial turmoil or regulatory change. Upon comparing the three procedures in terms of small sample power and estimation accuracy for the structural break date, they found that the recursive procedure performs best. Applying the recursive test of Quintos et al. (2001), Werner and Upper (2004) also demonstrated that the tail-index of Bund futures returns distribution exhibits structural break points. Furthermore, Candelon and Straetmans (2006) considered issues related to multiple structural changes occurring on unknown break dates. These authors applied the single break point recursive test devised by Quintos et al. (2001) in a sequential way, enabling the detection of gradual increases and decreases in tail-index, to test for multiple break points. Additionally, they showed the existence of multiple structural break points in the tail behaviour of emerging currency returns.

More recently, Lin and Kao (2008) proposed another empirical approach, built upon the work of Bai and Perron (1998) which specifies a time series of tail-indexes in a linear model with a multiple structural change framework, for detecting multiple structural changes of tail behaviour of the daily returns data for DJIA futures contracts.

In this paper we go one step further than the Lin and Kao (2008) analysis in that we propose a testing procedure for multiple breaks and apply it to American and French stock indexes data. Obviously, the numerous switches, financial and economic crisis over the recent history make financial market data the prime candidates for investigating the presence of multiple breakpoints. In particular, we attempt to explore the empirical evidence of the instability in tail behaviour, especially during financial crises, based on some developments in the analysis of structural change models. To that effect, we illustrate the applicability of the above-mentioned innovations using US stock indexes. In order to justify our applied procedures, we provide some experiment simulations and we find that they indicate good finite sample properties. The results indicate that some procedures perform reasonably well and lead to an appropriate number of breaks with locations coinciding with events and major financial crises such as the terrorist attacks in 2001 and the sub-prime crisis effect in 2008.

The rest of the paper is organised as follows. In section 2 we present a brief review on tail index estimation. The Bai-Perron (1998) approach for multiple breaks in regression parameters used for series of tail-indexes is discussed in Section 3. Section 4 then performs some simulation experiments that show if switching in the tail index is detectable. Empirical results are documented and discussed in Section 5. Section 6 offers concluding remarks.

2 Tail index

Let X_1, X_2, . . . be independent random variables with a common distribution function F which has a regularly varying tail

$$1-F(x)=x^{-\alpha}L(x),\ \ x\to\infty,\ \ \alpha>0 \tag{1}$$

where L is a slowly varying function satisfying $\lim_{t\to\infty}\frac{L(tx)}{L(x)}=1, \forall x>0$ and α is a shape parameter. This is the case if F is in the domain of attraction of an extreme value distribution with positive index or if F is in the domain of attraction of a stable distribution with index $0<\alpha<2$. Empirical studies on the tails of daily log-returns in finance have indicated that one frequently encounters values a between 3 and 4; see for instance Longin (1996) and Loretan and Phillips (1994). Various estimators for estimating $\gamma=\frac{1}{\alpha}$ have been proposed (see Hill, 1975; Pickands, 1975; de Haan and Resnick, 1980; Hall, 1982; Mason, 1982; Davis and Resnick, 1984; Csorgo et al., 1985, Hall and Welsh, 1985). We concentrate on the best known estimator, Hill's estimator,

$$\gamma_n(k)=\frac{1}{k}\sum_{i=1}^{k}\log X_{n,n-i+1}-\log X_{n,n-k} \tag{2}$$

where $X_{n,n-i+1}$ are the order statistics of X_1, ..., X_n. Further details are provided in Jansen and De Vries (1991) and the recent monograph by Embrechts et al. (1997). Hall (1982) showed for $k/n\to 0$ as, $n\to\infty$ that the statistic is asymptotically standard normally distributed. To determine m Goldie and Smith (1987) show that one picks m such that it is in a range that minimises the asymptotic Mean-Squared Error. Consequently, minimising the sample Mean Squared Error (MSE) is the appropriate selection criterion in large samples. A heuristic procedure for determining m constitutes in plotting the estimator as a function of m and selecting m in the region over which the estimator is more or less constant.

More statistically involved procedures have been proposed for small samples by Huisman et al. (1998, 2001). Their regression-based approach is based on an approximation of the asymptotic expected value of the Hill estimator as a linear function of k

$$E\left(\gamma_n(k)\right)\approx\frac{1}{\alpha}-ck \tag{3}$$

Here c is a constant depending on parameters of the distribution and the sample size. If k becomes small, the bias goes down and the expectation goes to the true value $\gamma=\frac{1}{\alpha}$. The variance of the estimator increases with small k

$$V\left(\gamma_n\left(k\right)\right) \approx \frac{1}{k\alpha^2} \tag{4}$$

The idea of Huisman et al. (1998, 2001) is to use equation (2) in a regression analysis and regress the γ(*k*) values (computed with an ordinary Hill estimator) against k as follows:

$$\gamma_n\left(k\right) = \beta_0 + \beta_1 k + \varepsilon\left(k\right),\ k = 1,\ldots,\kappa \tag{5}$$

The estimated $\widehat{\beta_0}$ is an estimator of $\gamma = \frac{1}{\alpha}$. The authors propose to choose κ = *n*/2 where n is the sample size.[1]

In this paper, we follow the method of Huisman et al. (1998, 2001), a modified version of the Hill estimator can be used to correct for the bias in small samples.

3 Bai and Perron's multiple structural break model

In this section, we summarise the main elements of the Bai and Perron approach[2] (BP) for estimating and testing linear models for multiple structural changes, focusing on the ones that are most relevant to our analysis in Section 5.

Bai and Perron (1998, 2003) suggest several testing procedures for single and multiple structural breaks when the dates of breaks are unknown. In this paper, the Bai and Perron (BP) method is used to estimate one or multiple structural breaks. The BP method has some interesting features. First, BP is one of a few methods that can deal with multiple structural breaks. Second, BP's method assumes that potential structural break points are unknown. This is important because structural break dates are not known in practice.

Consider the following structural change model with *m* breaks (*m* + 1 regimes), which allows for the change in mean on the level of the tail-index:

$$\gamma_t = \beta_j + u_t, \quad t = T_{j-1} + 1,\ldots,\ T_j \tag{6}$$

for *j* = 1,..., *m* + 1 and where we use the convention that $T_0 = 0$ and $T_{m+1} = n$. The break dates (T_1,..., T_m) are explicitly treated as unknown and for *j* = 1,..., *m*, we have $\lambda_j = \frac{T_j}{n}$ with $\lambda_1 < \ldots < \lambda_m$

In this model g is the tail-index of the returns distribution at time *t*, β_j is the k × 1 vector that represents the mean tail-index level in different regimes and u_t is the disturbance at time *t*.

The break points (T_1,..., T_m), are explicitly treated as unknown. The purpose is to estimate the unknown regression coefficients together with the break points when *n* observations on γ_t are available.

The goal is first to estimate the unknown coefficients $\beta = (\beta_1, \beta_2,\ldots, \beta_{m+1})$. The method of estimation used here was based on the least-squares principle. For each *m*-partition (T_1,..., T_m), the least squares estimates of β_j are generated by minimising the sum of squared residuals,

$$SSR\left(\beta_j\right) = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} \left(\gamma_t - \beta_j\right)^2 \tag{7}$$

Given the regression coefficient estimates based on a given m-partition (T_1,,..., T_m), which are denoted by $\widehat{\beta}_j(T_1,\ldots,T_m)$, where, where $\beta_j = (\beta_1, \beta_2,\ldots, \beta_{m+1})$. Substituting these into equation (7) the estimated breakpoints are given by

$$\left(\widehat{T}_1,\ldots,\widehat{T}_m\right) = \arg\min_{T_1,\ldots,T_m} SSR\left(T_1,\ldots, T_m\right) \tag{8}$$

where the minimisation is taken over all partitions (T_1,..., T_m) such that $T_i - T_{i-1} \geq [\varepsilon n]$. The break point estimators are thus global minimisers of the objective function. Finally, the estimated regression parameters are the associated least-squares estimates at the estimated m-partition $\left\{\widehat{T}_j\right\}$, $\hat{\beta} = \hat{\beta}\left(\left\{\widehat{T}_j\right\}\right)$.

A central result derived by Bai and Perron (1998) is that the break fraction $\hat{\lambda}_j = \frac{\hat{T}_j}{n}$ converges to its true value at the fast rate *n*, making the estimated break fraction super-consistent. Therefore we can estimate the rest of the parameters, which converge to their true values at rate $n^{1/2}$, taking the break dates as known.

3.1 Test statistics for multiple breaks

The determination of the existence of structural change and the selection of the number of breaks depends on the values of various tests statistics when the break dates are estimated. Bai and Perron discuss three types of tests: a test of no break versus a fixed number of breaks, a double maximum test and a sequential test.

First, they consider a sup*F* type test of no structural break ($m = 0$) versus $m = k$ breaks. The test is $\sup F_n(k) = F_n\left(\hat{\lambda}_1,\ldots,\hat{\lambda}_k; q\right)$, where $\hat{\lambda}_1,\ldots,\hat{\lambda}_k$ minimise the global sum of squared residuals (according to (8)). Next, the null hypothesis of no structural break against an unknown number of breaks given some upper bound is tested by double maximum tests. Bai and Perron consider two statistics, what they called the 'double maximum' statistics. The first double maximum statistic is given by

$$UD_{max} = \max_{1\leq m\leq M} supF_n\left(m\right), \tag{9}$$

where *m* is an upper bound on the number of possible breaks. The second double maximum statistic applies different weights to the individual tests such that the marginal p-values are equal across values of *m* and is denoted WD_{max} (see Bai and Perron 1998, for details).

Finally, they proposed a statistic, labeled $\sup F_n(1+1\backslash 1)$ for testing the null hypothesis of *l* breaks against the alternative hypothesis of *l*+1 breaks. This statistic is used to test whether the additional break leads to a significant reduction in the sum of squared residuals.

BP derive asymptotic distributions for the double maximum and $\sup F_n(1+1\backslash 1)$ statistics and provide critical values for various values of the trimming parameter (ε) and the maximum numbers of breaks (*M*).

Compared to other structural break tests, the BP method allows for general specifications when computing test statistics and confidence intervals for the break dates and regression coefficients. These specifications include autocorrelation and heteroskedasticity in the regression model residuals, as well as different moment matrices for the regressors in the different regimes.

3.2 Criteria for finding the number of breaks

Bai and Perron (1998, 2003) propose three methods to determine the number of breaks: the Bayesian Information Criterion, BIC (Yao, 1988), the Schwarz modified criterion, LWZ (Lui et al., 1997) and a sequential approach. In this paper, we advocate for the last method proposed by Bai and Perron.

The relevant procedure for estimating the number of breaks as suggested by Bai and Perron is based on the sequential application of the sup test using the sequential estimates of the breaks. First, examine the double maximum statistics to determine whether any structural breaks are present. If the double maximum statistics are significant, examine the sup $F_n(1+1\backslash 1)$ statistics to decide on the number of breaks, choosing the sup $F_n(1+1\backslash 1)$ statistic that rejects the largest value of l. Finally, the trimming parameter of at least 0.10 ($M = 8$) is recommended when allowing for heteroskedasticity and series correlation in the time series.

4 Simulation experiments

In this section, we investigate via simulations the performance of the above-mentioned tests considering various data-generating processes. We focus upon the sup F test's accuracy in determining single breakpoints in the presence of single or two breaks. The data generating processes follow a Generalized Pareto Distribution (GPD) with different tail index values. In Figure 1, we plot the densities of GPD for $\gamma = 0.8$ and $\gamma = 0.1$.

4.1 Data generating processes

We consider the following three data generating processes. The first process generates samples with a size of $n = 1500$. For the two remaining DGPs, the simulation involved generating samples with a size of $n = 2250$. For each process 1000 samples were drawn.

$$DGP_1 : \quad GPD(0.2, 0,1)_{(t=1\ldots750)} \Big| GPD(0.6, 0, 1)_{(t=751\ldots1500)}; \tag{10}$$

$$DGP_2 : \quad GPD(0.1, 0,1)_{(t=1\ldots750)} \Big| GPD(0.4, 0, 1)_{(t=751\ldots1500)} \Big| GPD(0.8, 0, 1)_{(t=1501\ldots2250)}; \tag{11}$$

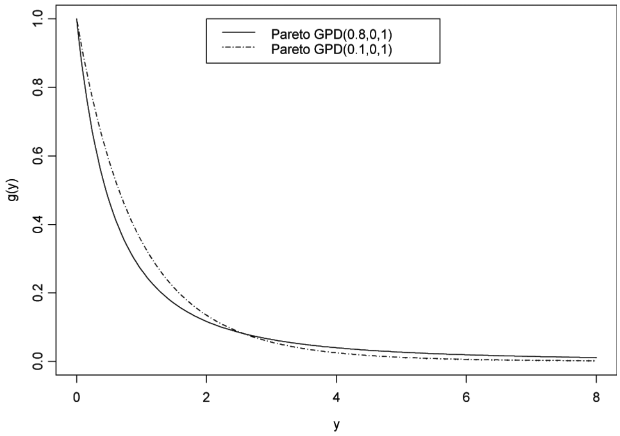
$$DGP_3 : \quad GPD(0.2, 0,1)_{(t=1\ldots750)} \Big| GPD(0.6, 0, 1)_{(t=751\ldots1500)} \Big| GPD(0.1, 0, 1)_{(t=1501\ldots2250)}; \tag{12}$$

The first process DGP_1 generates a series which follows a Generalised Pareto Distribution (GPD) in the 1rst half of the sample with $\gamma = 0.2$ and GPD distributed in the 2nd sample half with $\gamma = 0.6$. Both subsamples have equal length for sake of simplicity. The single break corresponds with a switch from thin tails with all moments in existence ($\alpha = 1/0.2 = 5$) to fat tails with tail index ($\alpha = 1/0.6 = 1.66$). The first process DGP_1generates an increasing tail-index series with one structural breaks T_1 (i.e., at time $t = 751$).

The second process generates series with a two breaks in g. The process implies an increase in$g$ followed by an increase ($g = 0.1$ in the first third of the sample, 0.4 in the second third and finally $\lambda = 0.8$). The DGP_2 generates an increasing followed by an increasing tail-index series with two structural T_1 (i.e., at time $t = 751$) and T_2 (i.e., at time $t = 1501$).

Finally, the third process generates also series with a two breaks in γ. The process implies an increase in γ followed by a decrease ($\gamma = 0.2$ in the first third of the sample, $\gamma = 0.6$ in the second third and finally $\gamma = 0.1$). In contrast to DGP_2, the DGP_3 generates an increasing followed by a decreasing tail-index series with twostructural T_1 (i.e., at time $t = 751$) and T_2 (i.e., at time t = 1501). As for DGP1 and DGP_2 all subsamples have equal length.

Figure 1 Generalised pareto densities, for pareto ($\gamma = 0.8$) and pareto ($\gamma = 0.1$)



The simulation investigation is organised as follows. This study begins by simulating the empirical Kernel distributions of the estimated break points to each proposed DGPs and finally studies the empirical power of the sup F type test statistic.

4.2 Estimating the break date

The simulation were conducted for a known break points location on $\tau^* = 751/1500 = 0.5$ (for DGP_1) and $\tau_1^* = 0.333$ and $\tau_2^* = 0.666$ (for DGP_2 and DGP_3). Table 1 reports the results of simulation experiments on the break point estimators $\hat{\tau}_{DGP1}$, $\hat{\tau}^{(i)}_{DGP2}$ and $\hat{\tau}^{(i)}_{DGP3}$ for I = 1,2. These values are compared to the true breakpoint τ^* and τ_i^* for i = 1,2 and are expressed as fraction of the sample size. The values in Table 1 are the mean values of the break point estimates. Below these values, in brackets, the corresponding empirical standard deviations are reported.

Table 1 Estimated break dates

Single break point: DGP_1				
	$\hat{\tau}_{DGP1}$			
$\tau^* = 0.50$	0.6117367 (0.0727564)			
Two break points: DGP_2 and DGP_3				
		$\hat{\tau}^{(i)}_{DGP2}$		$\hat{\tau}^{(i)}_{DGP3}$
$\tau_1^* = 0.333$	$\hat{\tau}_{DGP2}^{(1)}$	0.4588822 (0.1027592)	$\hat{\tau}_{DGP3}^{(1)}$	0.3759764 (0.03233747)
$\tau_2^* = 0.666$	$\hat{\tau}_{DGP2}^{(2)}$	0.7498996 (0.05245349)	$\hat{\tau}_{DGP3}^{(2)}$	0.5557582 (0.09967157)

Notes: The table reports mean values and standard deviations (in parentheses) of the break point estimators $\hat{\tau}_{DGP1}$, $\hat{\tau}^{(i)}_{DGP2}$ and $\hat{\tau}^{(i)}_{DGP3}$ for i = 1,2. These values as well as the true breakpoints τ^*, τ_1^* and τ_2^* are expressed as fraction of the sample size. Data is generated according to equations (10), (11) and (12).

From Table 1, we observe that the mean values of the estimates are not almost far off the true break point and the standard deviations are fairly low. When the DGP has a single point, $\hat{\tau}_{\text{DGP}\,1}$ turns out to be a reliable estimator. While, it tends to estimate the break point relatively late, the mean estimated break fraction is relatively closest to the true value and the standard deviation is relatively low. The proposed model with the sup *F* type test statistics thus can accurately estimate the break point for DGP_1. Similar, when the DGP has two break points, we note that in most of the case there is a tendency to estimate the break points too late. The only exception was found for $\hat{\tau}^{(2)}_{\text{DGP}\,3}$ which is relatively early.

The relatively late in estimating the break points date may be justified by the fact that the procedure takes some time for learning process before becoming effective in detecting break points. By comparing different DGPs, we note that the procedure tend to lately estimate the true breakpoint when we have an increasing tail index estimate and the opposite when we have a decreasing tail index estimate.

Similar results may be found by graphical representation of the derived Kernel densities based upon 1000 replications.

The relevant simulated Kernel densities are contained in Figures 2–4. Moreover, the frequency of the estimated break points are plotted against the relative location of the estimated break points in the sample (i.e., $T_j/n, j = 1, 2$). In Figure 2, the single break date for DGP_1 is accurately estimated by the supF test although slightly later than the true break (the peak in the frequency is around the 0.56 mode). The densities of the estimated break dates in Figure 3 suggest a first break at the 0.36 mode which indeed correctly signals the first break in DGP_2 and a second break around 0.74 % mode. Both of them are relatively later than the true break points. In Figure 4, we observe that the first break point of DGP_3 is correctly detected around the 0.35 mode, while it signals early the second break point (the peak in the frequency is around 0.50 mode).

Figure 2 Simulated Kernel density of the break points for DGP1 (see online version for colours)

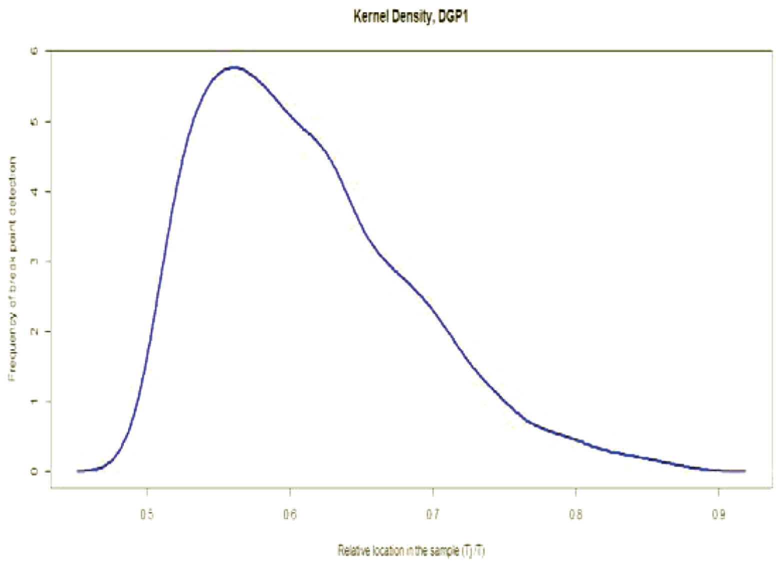


Figure 3 Simulated Kernel densities of the break points for DGP2 (see online version for colours)

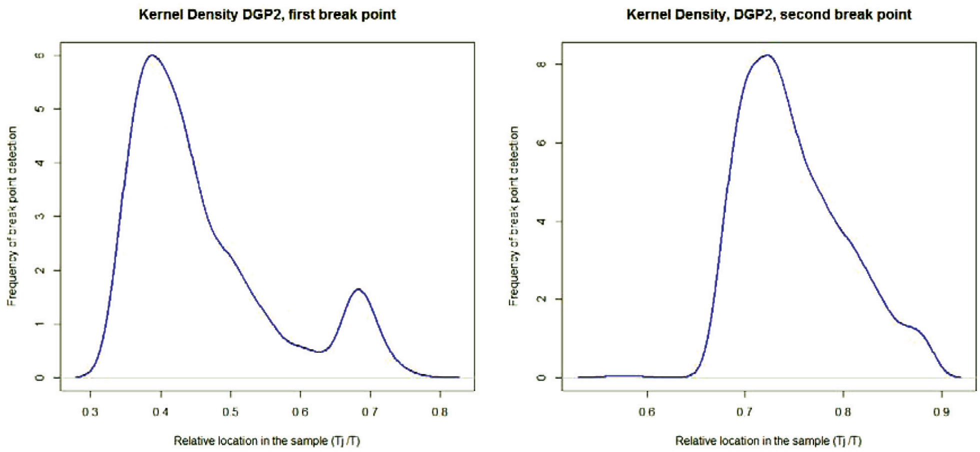
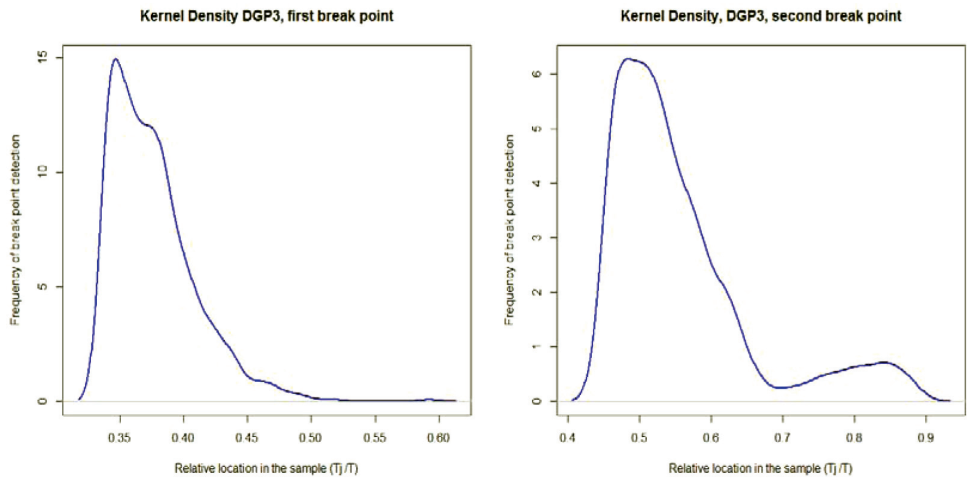


Figure 4 Simulated Kernel densities of the second break point for DGP3 (see online version for colours)



According to Table 1 and these figures, the proposed model with the sup F type test statistics thus can accurately estimate the break points for DGP1, DGP2 and DGP3, implying that it effectively identifies multiple structural changes for both increasing and decreasing true tail-index series, while it's in most of cases lately detected.

4.3 Power properties of sup F tests

We examine the extent to which the procedures find the right number of breaks when there are in fact some breaks. A measure of the test power is the probability of finding one or two breaks. We investigate the power of sup F tests to locate the breakpoint. To calculate the empirical power of the sup F test, we generate data according to (10), (11) and (12), 1000 replications are performed from each DGP. The power of the test is evaluated at a nominal size of 5%, i.e., a test rejects the null hypothesis. The results are reported in Table 2.

Table 2 Relative rejection frequencies of F-statistics

	Sup F
DGP_1	1.00
DGP_2	1.00
DGP_3	1.00

Note: sup F denotes the statistic sup F_n (k) for k = 1,2

Table 2 shows very good power properties of sup F tests; for both $k = 1$ and 2. The sup F test rejects the null hypothesis in 100% of the cases.

5 Empirical results

5.1 Data description

Our dataset consists of closing daily stock market indexes of USA and France. The data covers the period 01/02/1990 through 02/04/2012 for a total of 5592 usable observations for each index. This data set presents interesting cases for studying tail behaviour given the number and size of the macroeconomic shocks and financial crisis that occurred during this period, such as, the Tequila crisis in Mexico in 1994/1995, the Asian crisis in 1997/1998, the Russian default in 1998, the events of September 11, 2001, the Iraq war in 2003,the subprime mortgage crisis during 2008–2009 and the EU debt crisis triggered on 2010. Moreover, the retained period span is marked by large price increases and decreases that reflect a substantial rise in the volatility of main international stock market indexes.

Our objective is to implement tests of tail index constancy and, if possible, to relate the detected breakpoints to known changes in financial and economic area. The major crises shifts during the data period make them the obvious candidate for testing the multiple structural breaks in tail behaviour.

Daily returns are generated by taking first differences of the logarithm of each variable:

$$r_{i,t} = 100 * \ln\left(\frac{x_{i,t}}{x_{i,t-1}}\right),$$

where $x_{i,t}$ *s* the daily closing value of the stock market index *i* on day *t*. Figure 5 plots of the daily returns of both series and reveals that the largest positive and negative price movements occur around major financial crises. Both series were extremely volatile, which led to a succession of extremely large positive and negative returns within a very short time span. These graphics show that returns are stationary and suggestan ARCH scheme for the daily returns where large changes are followed by large changes and small changes are followed by small changes. Clearly, the degree of extreme return fluctuation varies before and after those major crises. One reason might be that the tail thickness of the underlying return distribution has changed over time.

Figure 5 Time series plot for CAC 40 and SP500 log returns series (see online version for colours)

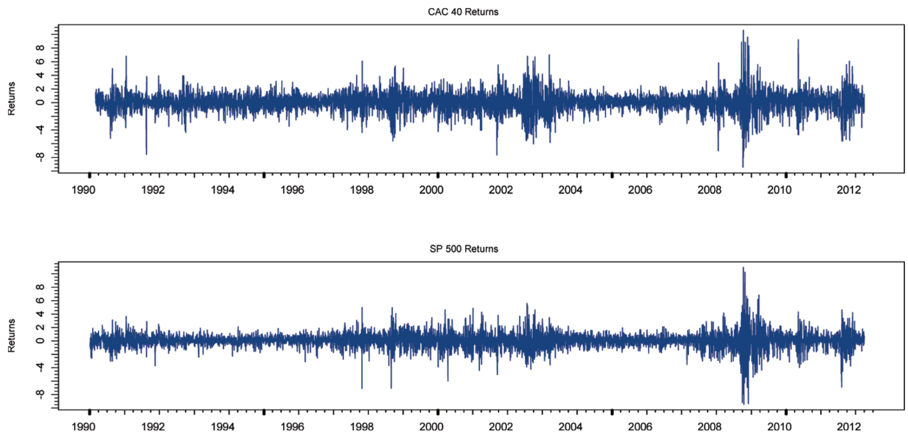


Table 3 provides data sample statistics. The mean return for the entire period is nearly zero. The SP500 exhibits the highest standard deviation. The unconditional distributions of the returns have a fat tail or are leptokurtic, as evidenced by high kurtosis, highly significant Jarque-Bera statistics. Additionally, the distributions have a slight negative skew, indicating a small asymmetry. These findings suggest that the distributions of daily returns can be characterized by a heavy tail or have a Fréchet tail distribution and the left and right tails should be treated separately.

Table 3 Descriptive statistics of daily returns

	SP500	*CAC40*
Observations	5992	5992
Mean	0.02447	0.01139
Std	1.181	1.436
Kurtosis	11.46	7.539
Skewness	–0.2323	–0.02893
Min (%)	–9.47	–9.472
Max (%)	10.96	10.59
Jarque-Bera	16762.2	4800.996
(p-value)	(0.00)	(0.00)

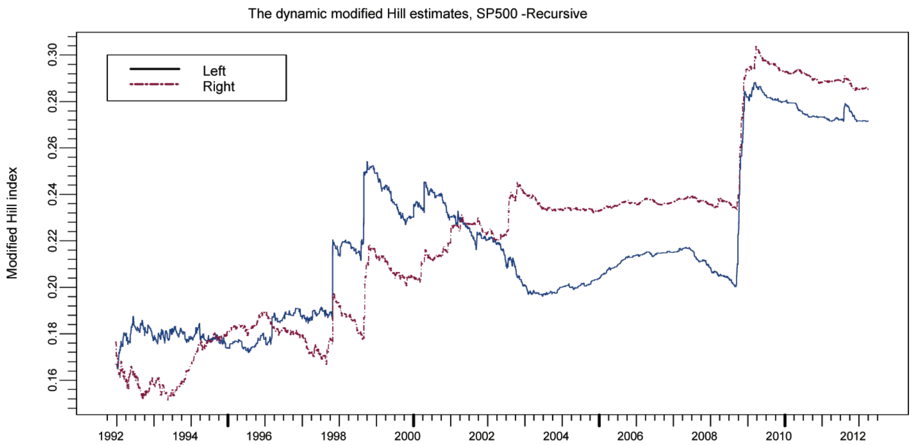
5.2 Tail-index estimates: a recursive approach

As we have noted, a simple graphical inspection of time series plot of returns shows that the degree of extreme returns fluctuation varies before and after major crises. One reason might be that the tail thickness of the underlying return distribution has changed over time. This finding let us motivated to assess the instability of the tail index.

To do that, we start our empirical analysis by calculating the recursive modified Hill[3] index of returns at time *t*, for both series, using of EVT techniques and then employs the structural changes model to estimate the unknown break date in a time series of tail-indexes and mean tail-index levels in different regimes.

By adopting a recursive approach, we start with a window size of 500 observations and increases the sample size by a single observation. Particularly, extreme price movements may differ between long and short positions. Consequently, one focuses in the first case on the left tail of the distribution of returns and on the right tail of the distribution in the second case. The resulting process of the evolution of the tail-index from March 1992 to July 2010, for SP500, is shown in Figure 6. This figure indicates that the modified Hill index[4] estimates (left and right) of the SP500 (CAC 40) returns distribution have mainly fluctuated in the range of about0.1512874 – 0.3037154 (0.186-0.275). These findings show that the distributions of daily returns can be characterized by a heavy tail or have a Frechet tail distribution. Even though tail-index differences between left and right tails do exist, the tail-indexes tend to move closely together.

Figure 6 The dynamic modified Hill index estimates for left and right tails of the SP500 returns distribution (see online version for colours)



It is clear from Figure 6 that both left and right indexes for SP500 exhibit a change in the degree of extreme movements over time. One source of such variation in tail activity in a series is that the tail thickness of the underlying returns distribution. Moreover, tail indexes for both series stay relatively stable around 0.18, until the mid 1990, followed by an upward trend indicating an increase in extreme movement in the markets during the periods, which are roughly about the time when many financial crises occurred such as crises in so-called emerging markets, the Tequila crisis in Mexico in 1994/1995, the Asian crisis in 1997/1998, the LTCM crisis of 1998, the Russian default in 1998, events of September 11, 2000, Irak war 2003. The tails seem to be particularly fat by the end of 2008 and later. This is not surprising, given that in that period, the market saw some of the worst turbulence in the international financial markets in living memory, the subprimes crisis during 2008. Moreover, the relatively high in the tail index during 2010 and later may be due the EU debt crisis.

Graphical investigation of the figure above show that the tail index is time varying or in others words the probability of extreme returns are not constant over time. This finding is a first sign of existing of multiple structural changes, namely, the probability of extreme events varies among different regimes, which we shall explore in the following subsection.

5.3 *Testing for Multiple Structural Changes using the Bai and Perron (2003) approach (BP)*

In this section, we use the method of Bai and Perron (2003) to test the presence of structural changes in the mean of the series of both tail indexes over the past 20 years. To that effect we apply our procedure with only a constant as regressor (i.e., $\{z_t = 1\}$) (inclusion of the lagged dependent variable does not lead to different results and was not found to have significant influence in the segmented model) and account for potential serial correlation via non-parametric adjustments (see the discussion in Section 4). We allowed up to 8 breaks and used a trimming $\varepsilon = 0.10$, hence each segment has at least 550 observations. This method requires no prior information regarding the number and timing of potential breaks and allows for serial correlation and heteroskedasticity in the errors across structural regimes.

BP considers several testing procedures aimed at identifying the number of structural breaks (m) in equation (1). In this section, we use two statistics developed by BP, what they called the 'double maximum' statistics, for testing the null hypothesis of no structural breaks against the alternative hypothesis of an unknown number of breaks given an upper bound M< (M = 8). The first double maximum statistic is given by UDmax and the second double maximum statistic applies different weights to the individual tests such that the marginal p-values are equal across values of M and is denoted WDmax (see Bai and Perron, 1998).

First, look at the sequential method. If 0 against 1 break is rejected, continue with the sequential method until the first failure to reject. If 0 against 1 is not rejected then test the hypothesis of no break versus a fixed number of breaks. If any sup F(k) (for = 0 versus = 1,…, *k* breaks, where k is the maximum number of breaks considered) is significant, then the number of breaks can be decided upon a sequential examination of the for = 1, …, *k* breaks. The number of breaks are decided by examining the Sup$F(l+1|l)$ statistics, choosing the $SupF(l+1|l)$ statistic that rejects for the largest value of l. Finally, the trimming parameter of at least 0.10 (M = 8) is recommended when allowing for heteroskedasticity and series correlation in the time series. The results are presented in Table 4.

The first issue to be considered is the determination of the number of breaks. Here one key observation is that for each tail series, both double maximum values (UDmax and WDmax) support rejecting the null hypothesis of no structural breaks. All UD max and WD max statistics are significant at the 1% level. Additionally, the $Sup\ F_n$ (k) tests are all significant for k between 1 and 8 for both series. So at least one break is present.

For the tail estimate of SP500, the sequential procedure (using a 5% significance level) selects 5 breaks for both the tailswhile the BIC and the modified Schwarz criterion of Liu et al. (1997) retain 6 breaks for both tails. In contrast, for the CAC40, we decide to retain respectively 7 and 3 for left and right estimates, as an optimal for the sequential procedure while the BIC and the LWZ lead respectively to 7 and 8. Given the documented facts that the information criteria are biased downward and that the sequential procedure performs better in this case, we conclude in favour of the presence of five and six respectively for left and right concerning SP500 estimatesand seven and three respectively for left and right estimates of CAC40 series.

Based on the optimal number of breaks found, Table 5 reports the estimates of break dates and different mean tail-index levels obtained under global minimisation.

Our results are relevant for at least two reasons. Firstly, we observe that the break dates are generally accurately estimated since the 95% confidence intervals cover a few days before and after. In addition, these results are significant as they fit great facts and financial and economic events. This is what is showed below:

Table 4 Bai and Perron Tests of Multiple Structural Breaks in the tails of SP500 and CAC40 returns distribution

	SP500		*CAC40*	
Test statistics	*Left tail*	*Right tail*	*Left tail*	*Right tail*
Double maximum tests				
UDmax[a]	291.0052*	983.0570*	52.7451*	230.1117*
WDmax[b]	579.1146*	1671.1352*	105.9589*	347.4370*
Sup *F* test				
Sup $F_n(1)$	14.5103*	47.3831*	8.1817*	21.8583*
Sup $F_n(2)$	38.6034*	99.1532*	37.4031*	36.3282*
Sup $F_n(3)$	240.4568*	164.5954*	30.804*	213.6782*
Sup $F_n(4)$	291.0052*	691.1288*	37.0006*	230.1117*
Sup $F_n(5)$	268.1874*	983.0570*	41.6388*	183.2935*
Sup $F_n(6)$	228.1762*	841.9189*	52.7451*	180.4582*
Sup $F_n(7)$	193.0236*	728.1458*	47.5693*	155.9357*
Sup $F_n(8)$	227.8275*	657.4356*	41.6849*	136.6840*
Sup $F(l+1\|l)$ tests				
Sup $F_n(2\|1)$	21.1740*	509.8983*	37.4048*	1640.9524*
Sup $F_n(3\|2)$	44.3084*	67.9447*	4.4458	40.2483*
Sup $F_n(4\|3)$	1047.9490*	536.0893*	9.6759	5.5014
Sup $F_n(5\|4)$	14.0161*	536.0893*	10.3891	5.9963
Sup $F_n(6\|5)$	0.5873	3.2417	28.6790*	5.9963
Sup $F_n(7\|6)$	0.0000	0.0537	19.9472*	5.9963
Sup $F_n(8\|7)$	0.0000	0.0000	0.0768	0.0000
Sequential	5.0000	5.0000	7.0000	3.0000
LWZ	6.0000	6.0000	7.0000	8.0000
BIC	6.0000	6.0000	7.0000	7.0000

Note: The upper bound M is set to be 8 and the trimming percentage is chosen to be 10% in the empirical study. The number of breaks chosen is according to the test statistics Sup $F(l+1|l)$, $l = 1, 2, 3, 4,5,6,7$.

The double maximum statistics (UDmax and WDmax) are highly significant, indicating that there is at least one structural break in the time series. The number of breaks are decided by examining the *Sup* $F_n(l+1|\ l)$ statistics, choosing the, *Sup* $F_n(l+1|\ l)$ statistic that rejects for the largest value of l.

*Significant at the 1% level.

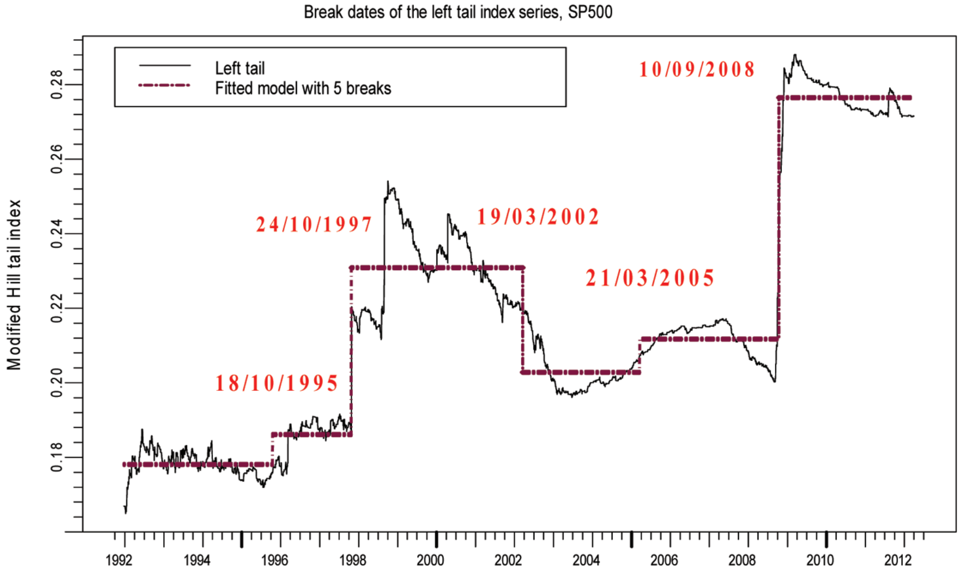
For the SP500 return distribution, the five estimated break dates for the left tail are October18, 1995, October 24, 1997, March 19, 2002, March 21, 2005 and September 10, 2008, their 95% confidence intervals are (15/07/1995–30/05/1996), (08/08/1997–04/11/1997), (17/01/2002 –30/09/2002), (03/06/2003–14/03/2007) and (01/09/2008–23/09/2008), respectively (see Figure 7). Their estimates of the mean tail-index levels in the six regimes are 0.178088, 0.186101, 0.230882, 0.202843, 0.211730 and 0.276542, respectively, values which are significantly different from 0 at the 1% level. Note that the differences in the estimated means over each segment are significant and point to an increase of 4,50% in October 18, 1995, another increase of 24,06% October 24, 1997, a decrease of 12,14% in March 19, 2002, an increase of 4,38% in March 21, 2005 and a large increase of 30,61% in September 10, 2008. These variations of the estimated autoregressive coefficient across segments indicate that the effects of some international economic and financial events. Additionally, our results show

Table 5 Estimation results of the structural change model in the tails of SP500 returns distribution

		SP500	
		Left tail	*Right tail*
1st regime	δ_1	0.178088*	0.159380*
	First break date	18/10/1995(15/07/1994–30/04/1996)	01/02/1994 (15/10/1993–14/03/1994)
2nd regime	δ_2	0.186101*	0.180714*
	Second break date	24/10/1997 (08/08/1997 – 04/11/1997)	04/09/1998 (04/08/1998–30/10/1998)
3rd regime	δ_3	0.230882*	0.209819*
	Third break date	19/03/2002 (17/01/2002 - 30/09/2002)	04/12/2000 (12/10/2000–07/03/2001)
4th regime	δ_4	0.202843*	0.228069*
	Fourth break date	21/03/2005 (03/06/2003–14/03/2007)	19/12/2002 (09/09/2002–22/03/2004)
5th regime	δ_5	0.211730*	0.237148*
	Fifth break date	10/09/2008 (01/09/2008–23/09/2008)	10/09/2008 (05/09/2008–28/09/2008)
6th regime	δ_6	0.276542*	0.291176*

that there are confidence intervals which are large, indicating that the estimates are imprecise and so no evidence in support of structural changes occurring on October 18, 1995 and on March 21, 2005.

Figure 7 The time series plot of the left tail-index of the SP500 returns distribution with estimated five break dates and six mean tail-index levels in different regimes (see online version for colours)



Our empirical study shows an increase in extreme negative price movements for SP500 returns the third (after mid 1990s – beginning 2002) and the six regimes (since the last quarter 2008) compared with the others regimes, with an increasing and decreasing in the level of the estimates of tail index.

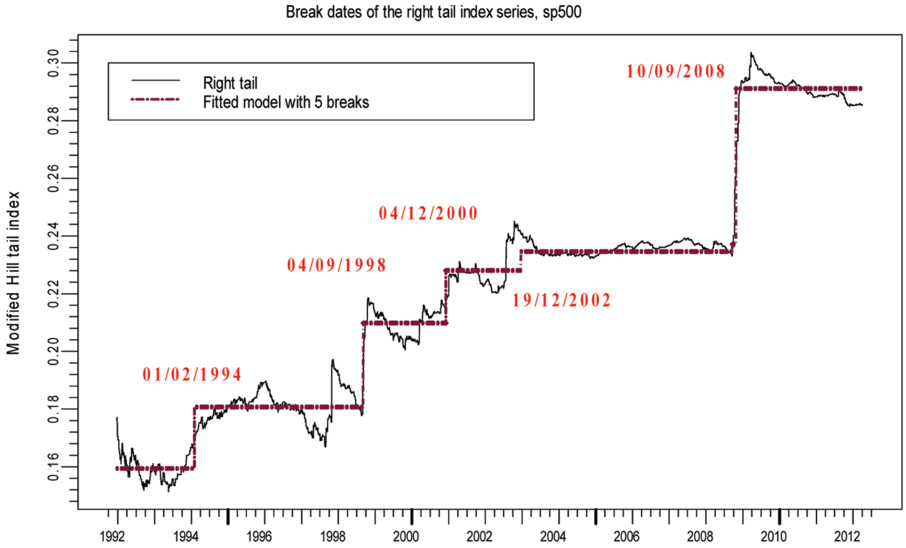
These empirical findings show that high levels in the estimated mean tail-index, generally coincide with periods of particular financial turbulence in the US and international markets. In fact, equity price volatility has trended up since the mid-1990s as it is affected by some catastrophic financial crises in so-called emerging markets, the Tequila crisis in Mexico in 1994/1995, the Asian crisis in 1997/1998, the Russian default in 1998, the crisis in Brazil the same year, the slide into crisis and eventual devaluation and default in Argentina in 2001. Equity volatility has been particularly high since 2000. The stock market began to fall from spring 2000 and then, more definitively, from late summer 2000, when a seemingly endless run of dismal corporate profit reports dramatically deflated equity prices. A huge multitude of e-commerce firms that had never shown a profit collapsed first, as they simply ran out of funds. But, soon the crash consumed almost all of the leading lights of the TMT sector (technology, media and telecommunications), during the dot.com bubble, followed by shocks such as the events of September 11, 2001, the Enron and WorldCom accounting scandals and geopolitical uncertainty, all of which conspired to keep equity markets falling. In late 2002, the Fed lowered interest rates and the economy slowly recovered during 2003, thus decreasing the extreme negative price movements. The last analysis of market conditions provides further evidence of a structural change in left tail behaviour occurring onMarch 19, 2002 for SP500 returns.

Note that the highest level in the mean tail-index estimate matches with the recent subprime crisis. By the end of 2008, the difficulties associated with the subprime mortgage lending began to spread to the broader financial sector, resulting in a second bear market of the 21 st century. The crisis escalated in September, 2008 entered a period of unusual volatility. A very violent fall intervened in October, 2008, just when this planetary cycle began its first true offensive. The prices were violently fired downward, sometimes by falling to the half of their value. In this year, the current loss of the largest since 1931 was reached, when the broader market declined more than 50%. The market continued to decline from late 2008 and early 2009 surrounding the events related to the financial crisis in 2008. The concomitant slowdown in the finance industry and the real estate market may have been a contributing cause of the 2008–2009 economic recessions. Additionally, EU debt crisis triggered on 2010 may explain the relative increasing level of risk observed in recent last years.

The prevailing uncertainty has been stoked by mounting geopolitical tension, increased caution regarding the financial information disclosed by companies and heightened risk aversion on the part of investors; it led to a significant and protracted drop in prices as well as a sharp increase in volatility on stock and credit markets. All of these events and the associated uncertainties leading up to keep the equity markets falling.

For the right tail-index of SP500, we also retain 5 break dates. The break dates are February 1, 1994, September 4, 1998, December 4, 2000, December 19, 2002 and September 10, 2008 (see Figure 8), their 95% confidence intervals are (15/10/1993–14/07/1994), (04/08/1998–30/10/1998), (12/10/2000–07/03/2001), (09/09/2002–22/03/2003) and (10/09/2008–28/09/2008), respectively. Their estimates of the mean tail-index levels in the six regimes are 0.159380, 0.180714, 0.209819, 0.228069, 0.237148 and 0.291176 respectively. The differences in the estimated means over each segment are significant and point to an increase of 13,39%

Figure 8 The time series plot of the right tail-index of the SP500 returns distribution with estimated five break dates and six mean tail-index levels in different regimes (see online version for colours)



in February 1, 1994, another increase of 16,11%in September 4, 1998, an increase of 8,70%in April 4, 2000, an increase of 3,98 % in December 19, 2002 and a large increase of 22,78 % in September 10, 2008.The results can also be visualised as in Figure 7. In contract, the right tail index shows an increase of probability of extreme positive price movement across the seven regimes. Note that there is confidence interval which is large, indicating that the estimate is imprecise and so no evidence in support of structural changes occurring on February 1, 1994.

In contrast to negative extremes returns, the right tail index shows an increasing probability of extreme positive price movement across the six regimes. Note that the highest estimate of the mean tail-index was on the last regime which matches with the sub-prime crisis of 2008. The empirical evidence indicates that the structural changes in the tail behaviour of the distribution of SP500 returns are associated more with negative shocks than positive ones, creating differences in risk management between long and short investors in markets. Additionally, these break dates almost all occur at the end of the year.

Note that for the CAC index and in order to keep the paper short, we present only a summary of the results, tables and graphs are not reported and are available upon request from the authors. The eight estimates of the mean tail-index levels corresponding to the negative tail are 0.246859, 0.221706, 0.229807, 0.212650, 0.224851, 0.226936 and 0.246909, respectively, values which are significantly different from 0 at the 1% level. The differences in the estimated means over each segment are point to a decrease 10,19% % in June 23, 1994[5] (22/06/1994–27/06/1994), an increase of 3,7% in July 27, 1998 (15/07/1998–03/08/1998), a decrease of 7,5 % in July 28, 2000 (27/07/2000–01/08/2000),an increase of 5,7% in July 31, 2002 (30/07/2002–02/08/2002), a slight increase of 0,9% in September 9, 2004 (09/09/2004–06/10/2004) and an important increase of 8,8% in September 16/09/2008 (08/09/2008–10/10/2008). Break dates are generally linked to major events in the market. For CAC 40, break dates on September 1998, July and September 2002 and October 2008 are easily justified. The period of trouble is identified at the end of 1998 and the beginning of 1999. This may be linked with the implementation of the European single currency and the risk born on the market through this event. Another period of trouble occurs in 2000. This seems to be a consequence of the IT 2000 bubble burst. Fourthly, period of trouble and the longer one, is witnessed between the end of July 2002 and April 2003. It may be a consequence of the Iraqi War II and the threats about it. The US subprime housing market started its downward trend at the end of 2007 and Lehman Brothers declaration for bankruptcy the 15 of September 2008, caused a dramatic effect on the US stock markets that rapidly spread to rest of the world, especially on French stock market. Additionally, we can note that other domestic or macroeconomic factors may explain the existence of other switching regime on French stock market.

Compared to the SP500, the estimates of mean tail-index levels seem more stable and are maintained in the range 0.21–0.24. The highest levels are observed on in June 23, 1994 andin 16/09/2008. Note that the left tail index break dates for SP 500 and CAC are dissimilar (dates and years) except for Sep. 10, 2008 for SP 500 and Sep. 16, 2008 for CAC which are close enough. Such disparity in break dates for the two indices may tentatively be interpreted by the existing of some country-specific effects, such as macroeconomic factors that may partly drive stock market movements. However, when there is a strong crisis such as the subprime crisis of 2007–2008 both of them are more efficient in integrating the news, exhibiting a more resilient stout behaviour and the switching is being more obvious and occur in almost the same time.

The estimated break dates for the right tail are in Mars 15, 1994, October 10, 1998 and September 16, 2008. Their estimates of the mean tail-index levels in the six regimes are 0.229913, 0.197534, 0.237069 and 0.268043. The differences in the estimated means over each segment are point to a decrease of 14,08% in Mars 15, 1994 (17/02/1994–07/11/1994), an increase of 20,01% in October 10, 1998 (12/08/1997–03/01/1998) and an increase of 13,07 % in September 18, 2008 (08/09/2008–10/10/2008). The highest levels are observed on in Mars 15, 1994 and in September 16, 2008.

Note that the highest level is relevant to period of subprime crisis of 2008. In fact, the deterioration in the economic and financial environment has seen the stock market index at new lows. In such a climate of extreme anxiety, the stock market indices reached levels close to our worst-case scenario. Investors are well aware of the country's weaknesses and of the poor state of the French economy, something it has in common with many countries. Our results show that French stock market moves broadly in line with USA especially in very strong crisis such as the last subprime crisis. In such violent crisis where volatility was extraordinarily high, this co-movement is very obvious and therefore the switching in the tail behaviour of stock indexes can be easily observed and nearly on the same period.

In sum, the recent financial crisis has triggered large adverse effects on stock markets around the world, especially in American and European countries. However, the degree of extreme risk seems not uniform. The US market has higher risk. However, for both market, the most significant jumps are clearly observed in September 2008.

Conclusion

This study is motivated by our aim to further explore the empirical evidence in the tail behaviour returns of stock market indices, based on some developments in the analysis of structural change models, in turbulent periods. To do that, we have used some innovations related to structural change that can simultaneously determine the number of structural breaks in a series of tail-indexes and estimate the mean tail-index levels in different regimes. Here we have used a commonly accepted and practiced estimator for the tail index, Hill estimator.

Based on some simulation exercises, we examine the small-sample performance of the procedure suggested by Bai and Perron for determining the true location of multiple breaks, based on the sequential Sup F test and we provide some results on the power of the test, that indicate good finite sample properties of our procedure. The proposed method is then applied to the tail behaviour returns of two international stock market indices, S&P500 (USA) and CAC40 (France), covering the period October 1990 to July 2010. This data set is challenging to model given the number and size various recent financial crises, during this period. Our results confirm the instability in the tail behaviour of return distributions and indicate that applied procedures perform reasonably well and lead to an appropriate number of breaks with locations coinciding with major financial crisis and events such as the LTCM crisis, September 11, 2001 terrorist attack on the US, the sub-prime crisis by the end of 2008 and the EU debt crisis triggered on 2010.

The instability of the tail index has important implications for the design of trading and risk management models. In fact, if the amount of probability mass in the tails of the unconditional distribution is shifting through time, the full sample estimates of tail index and corresponding quantiles, like Value at Risk, might wrongly estimated.

Our paper is a contribution in exploring and modeling time variation of tail behaviour, based on some developments in the analysis of structural change models. However, the results we have presented show the need for more investigation and therefore can be extended in several ways. We studied the tail behaviour of stock indexes and we found that can vary significantly across regimes. In future research, one could study the implications of the instability in tail behaviour on the Value-at Risk or others risk measures. Such extension is likely to deepen our understanding of the effects of multiple regimes on risk management strategies or on asset allocation. Finally, one could extend our framework to jointly modeling regimes in tails behaviour of a set of stock market indices as they are well-known closely linked and generally move together, especially in turbulent periods.

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Notes

[1]The authors show that the results are robust with respect to the choice of κ.

[2]Among the existing literature on the subject, see Chow (1960), Quandt (1960), Andrews (1993) and Andrews and Ploberger (1994). More recently, we find the work Bai and Perron (1998, 2003).

[3]The modified Hill index is calculated following the method proposed by Huisman et al. (2001) and Quintos et al. (2001).

[4]Note that a high value of $\hat{\gamma}$ implies thicker tails.

[5]The dates in parentheses are the confidence intervals corresponding to the break dates.